Compton Scattering →

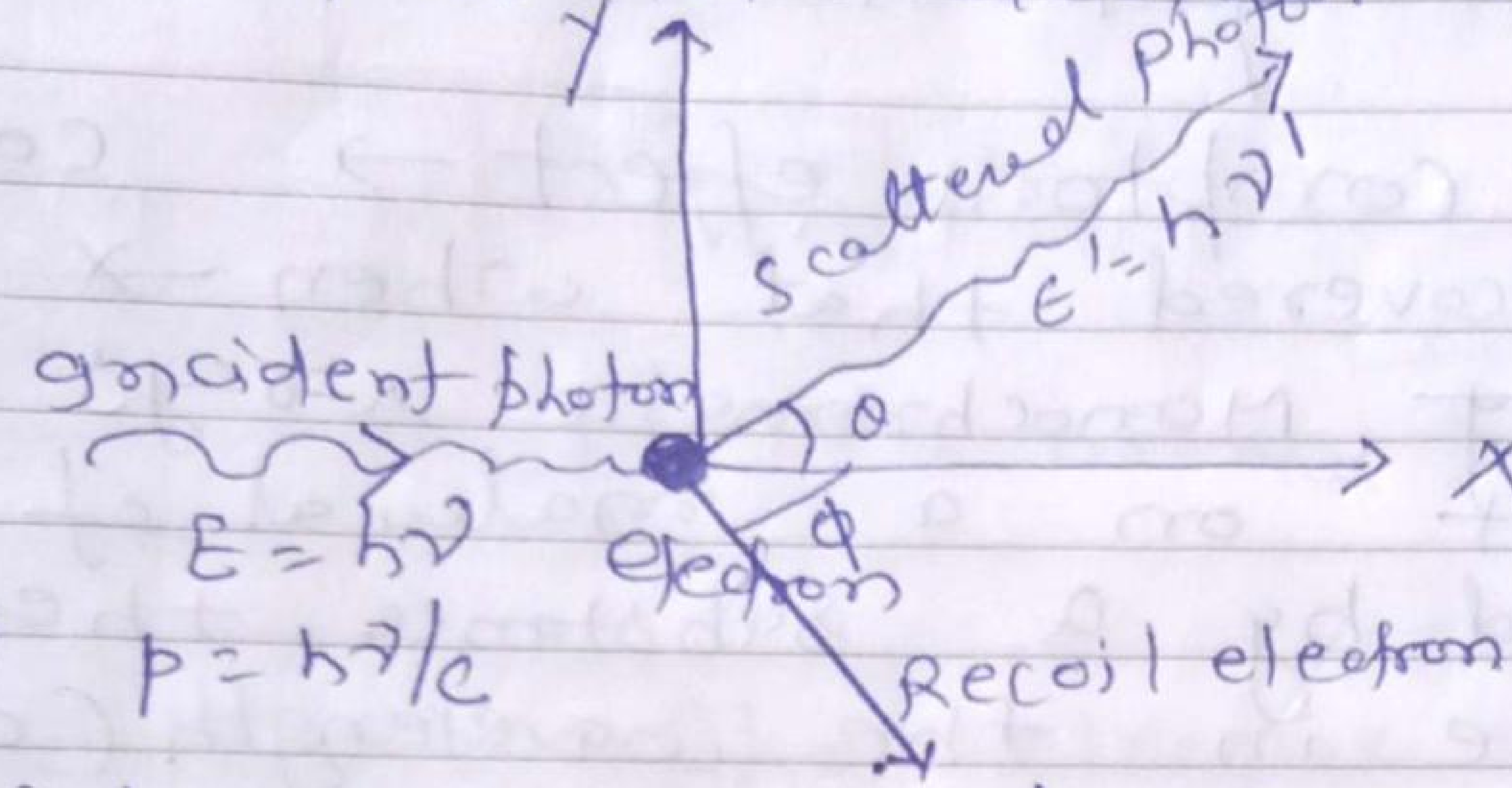
The compton effect → compton discovered that when ~~x-rays of a sharply~~ Monochromatic x-rays were incident on a material ~~of~~ they scattered by a substance, there is an increase in the wavelength (decrease in frequency) of scattered x-rays and this increase in the wavelength depends upon the angle of scattering. This ~~pheno~~ phenomenon is called the compton effect and the difference between the wavelengths of the modified radiation and the unmodified radiation is called compton shift.

Explanation → ~~This~~ compton effect explained on the basis of quantum theory of radiation. The whole process is treated as a particle collision event between x-ray photon and a loosely bound electron of the scatterer. In this process momentum and K.E are conserved (elastic collision). ~~In this process, both momentum and energy are conserved~~ In the photon electron collision, a portion of the energy of the photon is transferred to the electron so the x-ray proceeds with less than the original energy.

The incident photon with an energy $h\nu$ and momentum $h\nu/c$

M	T	W	T	F	S	S
30	31					1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29

strikes an electron at rest.



fig(a) p = hν/c

Initial momentum of electron = 0

Initial energy due to rest mass energy = m_0c^2

The scattered photon of energy = $h\nu'$

momentum = $h\nu'/c$

~~Moves~~ The scattered photon moves of in a direction incident at an angle ~~to the~~ original direction = θ

The electron acquires

momentum = mv

moves at angle = ϕ with original direction

energy = mc^2

According to principle of conservation of energy

$$h\nu + m_0c^2 = h\nu' + mc^2$$

$$h(\nu - \nu_0) = (m - m_0)c^2$$

According to conservation of momentum

$$h\nu/c = h\nu'/c \cos\theta + mv\cos\phi$$

X ~~ct~~ component of momentum of scattered & recoil electron

Y component of momentum conservation

$$0 = \frac{h\nu'}{c}\sin\theta - mv\sin\phi$$

$$\frac{h\nu'}{c}\sin\theta = mv\sin\phi$$

$$h\nu'\sin\theta = mvc\sin\phi \quad (A)$$

$$\frac{h\nu}{c} = \frac{h\nu'}{c}\cos\theta + mv\cos\phi$$

$$(h\nu - h\nu'\cos\theta) = mvc\cos\phi \quad \text{———} \quad (B)$$

squaring and adding eqn (A) & (B)

$$(h\nu'\sin\phi)^2 + (h\nu - h\nu'\cos\theta)^2 = m^2v^2c^2(\sin^2\phi + \cos^2\phi)$$

$$h^2\nu'^2\sin^2\phi + h^2\nu^2 + h^2\nu'^2\cos^2\phi - 2h^2\nu\nu'\cos\theta = m^2v^2c^2$$

$$h^2\nu'^2(\sin^2\phi + \cos^2\phi) + h^2\nu^2 - 2h^2\nu\nu'\cos\theta = m^2v^2c^2$$

$$h^2\nu'^2 + h^2\nu^2 - 2h^2\nu\nu'\cos\theta = m^2v^2c^2 \quad \text{———} \quad (C)$$

$$mc^2 = h(\nu - \nu') + m_0c^2$$

squaring it

$$\{mc^2\}^2 = \{h(\nu - \nu') + m_0c^2\}^2$$

$$m^2c^4 = h^2(\nu^2 + \nu'^2 - 2\nu\nu') + 2h(\nu - \nu')m_0c^2 + m_0^2c^4 \quad \text{—} \quad (D)$$

Substracting (D) & (E)

$$m^2c^2(c^2-v^2) = -2h^2\nu'\nu(1-\cos\theta) + 2h(\nu-\nu')m_0c^2 + m_0^2c^4$$

$$m = \frac{m_0}{\sqrt{1-v^2/c^2}} \qquad m^2(1-v^2/c^2) = m_0^2$$

$$m^2(c^2-v^2) = m_0^2c^2$$

So

$$\cancel{m_0^2c^4} = -2h^2\nu\nu'(1-\cos\theta) + 2h(\nu-\nu')m_0c^2 + \cancel{m_0^2c^4}$$

So

$$+\cancel{2}h^{\cancel{2}}\nu\nu'(1-\cos\theta) = \cancel{2}h(\nu-\nu')m_0c^2$$

$$\frac{\nu\nu'}{\nu-\nu'} = \frac{m_0c^2}{h(1-\cos\theta)}$$

So

$$\frac{\nu-\nu'}{\nu\nu'} = \frac{h}{m_0c^2}(1-\cos\theta)$$

$$\frac{1}{\nu'} - \frac{1}{\nu} = \frac{h}{m_0c^2}(1-\cos\theta)$$

$$c/\nu' - c/\nu = \frac{h}{m_0c}(1-\cos\theta)$$

$$\boxed{\lambda' - \lambda = \frac{h}{m_0c}(1-\cos\theta)}$$

$\therefore d\lambda = \lambda' - \lambda =$ change in wavelength

$d\lambda = h/m_0c\,(1-\cos\theta)$ · (C ~~Compton shift~~)

$d\lambda$ depends upon angle of scattering only & does not depend upon wave length of incident radiation & nature of scattering substance.

(1) case I

If $\theta = 0$ $\cos\theta = 1$

$d\lambda = 0$

(2) case II

If $\theta = 90^\circ$ $\cos\theta = 0$

$$d\lambda = h/m_0c = \frac{6.63 \times 10^{-34}}{(9.11 \times 10^{-31}) \times (3 \times 10^8)} = 0.0243$$

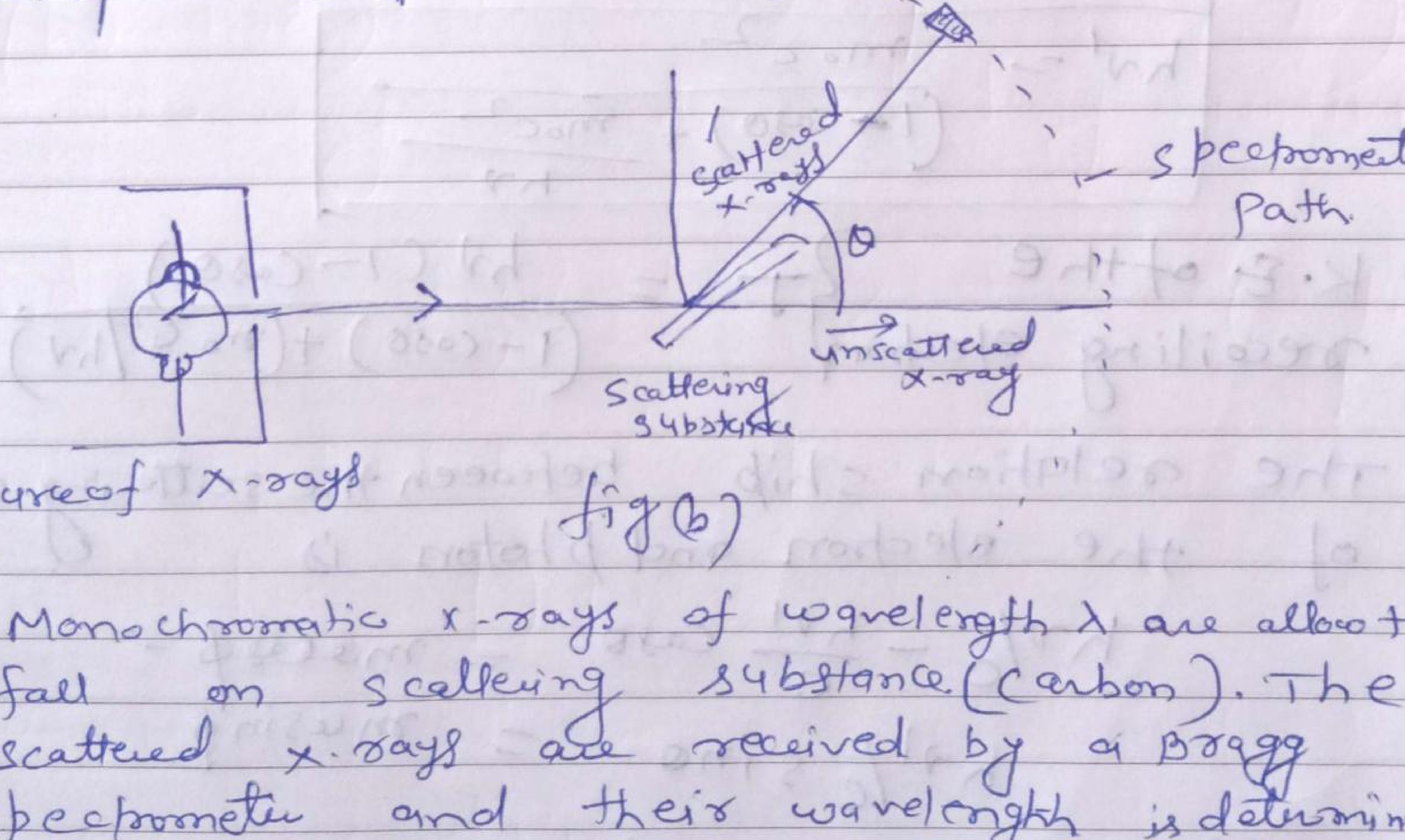
This is known as compton wavelength.

(3) case III

If $\theta = 180^\circ$ $\cos\theta = -1$

$d\lambda = 2h/m_0c = 0.0486 \text{ AU}$ (Max)

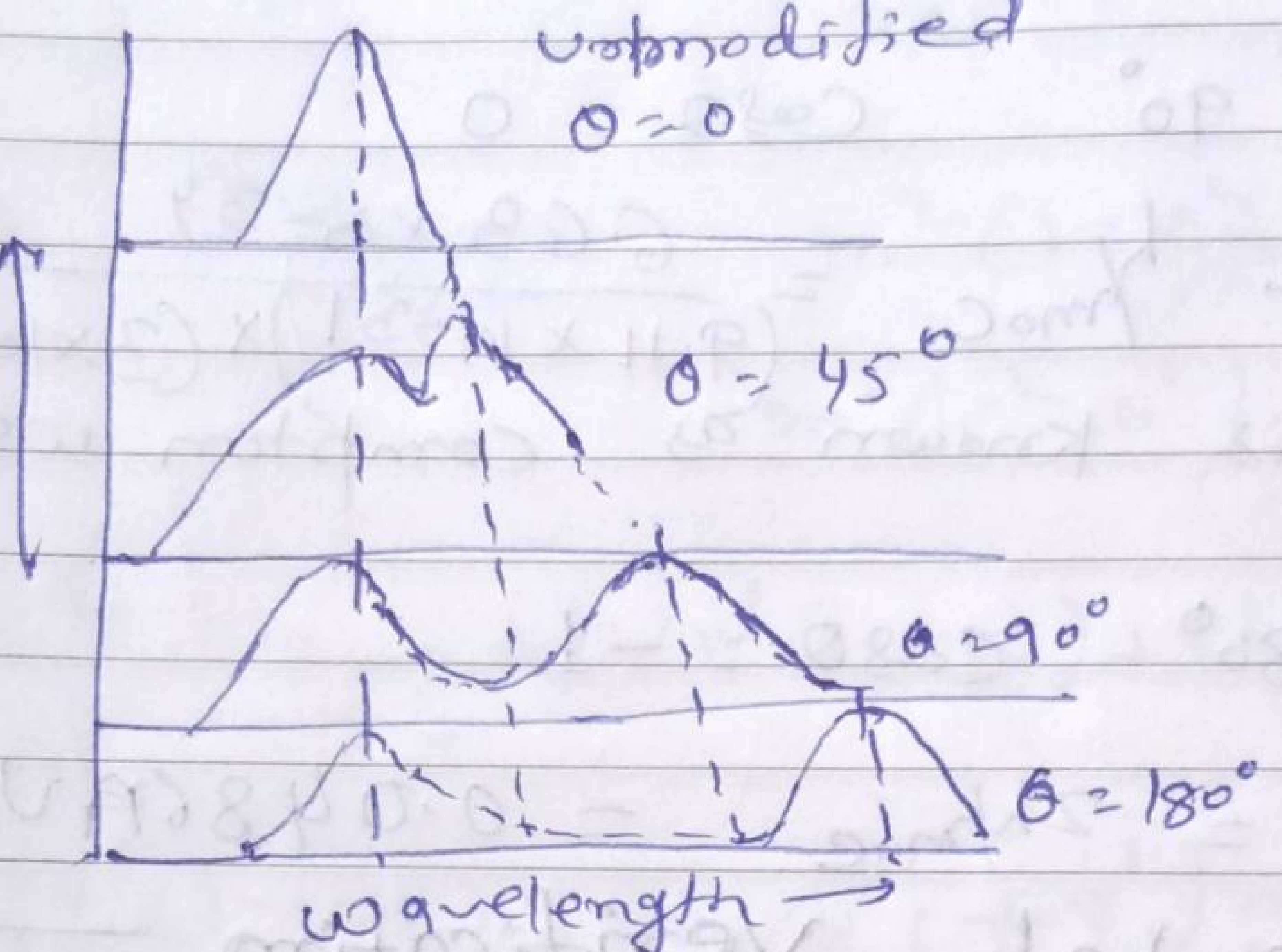
Experimental Verification →



fig(b)

Monochromatic x-rays of wavelength λ are allow to fall on scattering substance (carbon). The scattered x-rays are received by a Bragg spectrometer and their wavelength is determined.

The wavelength of the scattered x-rays is measured for different value of scattering angle shown in the fig(c). In the scattered radiation in addition to the incident wavelength there exists a line of longer wavelength λ_1. The "compton shift", $d\lambda$ is found to vary with the angle at which the scattered rays are observed



$$h\nu' = \frac{m_0c^2}{(1-\cos\theta) + \frac{m_0c^2}{h\nu}}$$

K.E of the recoiling electron } $T = \frac{h\nu(1-\cos\theta)}{(1-\cos\theta) + (m_0c^2/h\nu)}$

The relationship between the scattering angles of the electron and photon is

$$h\nu/c - \frac{h\nu'}{c}\cos\theta = mv\cos\phi$$

$$h\nu'/c \sin\theta = mv\sin\phi$$

So

$$\frac{mv\cos\phi}{mv\sin\phi} = \frac{h/c\,(\nu - \nu'\cos\theta)}{h\nu'/c\,\sin\theta}$$

$$\cot\phi = \left(1 + \frac{h\nu}{m_0c^2}\right)\left(\frac{1-\cos\theta}{\sin\theta}\right)$$

$$\cot\phi = \frac{(\nu/\nu' - \cos\theta)}{\sin\theta}$$